All Integration Techniques
Worksheet #1

1. $\int 5x\sqrt{3x^2+5}\,dx$

$u = 3x^2+5$ $\qquad \int 5x\, u^{1/2} \cdot \frac{du}{6x}$

$\frac{du}{dx} = 6x$ $\qquad \frac{5}{6}\int u^{1/2}\,du$

$dx = \frac{du}{6x}$ $\qquad \frac{10}{18}u^{3/2} + C$

$\frac{5}{9}(3x^2+5)^{3/2} + C$

2. $\int \frac{5x^3 + 4x^{2/3} - 2x\sqrt[3]{x}}{3x^{1/2}}\,dx$

$\frac{1}{3}\int (5x^3 + 4x^{2/3} - 2x^{4/3})\,x^{-1/2}\,dx$

$\frac{1}{3}\int 5x^{15/6} + 4x^{1/6} - 2x^{5/6}\,dx$

$\frac{1}{3}\left[\frac{30}{21}x^{21/6} + \frac{24}{7}x^{7/6} - \frac{12}{11}x^{11/6}\right] + C$

3. $\int \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$ $\qquad \int \frac{\cos u}{\cancel{x}} \cdot \cancel{x}\,du$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad \int \cos u\,du$

$dx = x\,du$ $\qquad \sin u + C$

$\sin(\ln x) + C$

4. $\int \ln(2x)\,dx$

$u = \ln(2x)$ $\qquad \int dv = \int dx$

$\frac{du}{dx} = \frac{2}{2x}$ $\qquad v = x$

$du = \frac{1}{x}\,dx$

$x\ln(2x) - \int \frac{1}{\cancel{x}} \cdot \cancel{x}\,dx$

$x\ln(2x) - x + C$

5. $\int \frac{-2x-19}{x^2-x-6}\,dx \rightarrow \frac{A}{(x-3)} + \frac{B}{(x+2)}$

$-2x-19 = A(x+2) + B(x-3)$

$x=-2:$ $\quad$ $x=3:$

$-15 = -5B$ $\quad$ $-25 = 5A$

$B=3$ $\quad$ $A=-5$

$\int \frac{-5}{x-3} + \frac{3}{x+2}\,dx$

$-5\ln|x-3| + 3\ln|x+2| + C$

6. $\int e^{3x+2}\,dx$

$u=3x+2$ $\quad$ $\int e^u \cdot \frac{du}{3}$

$\frac{du}{dx}=3$ $\quad$ $\frac{1}{3}\int e^u\,du$

$dx=\frac{du}{3}$ $\quad$ $\frac{1}{3}e^u + C$

$\frac{1}{3}e^{3x+2} + C$

7. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(2x)\,dx$

$u=2x$ $\quad$ $\int_{\pi}^{3\pi} \sin u \frac{du}{2}$

$\frac{du}{dx}=2$ $\quad$ $\frac{1}{2}\int_{\pi}^{3\pi} \sin u\,du$

$dx=\frac{du}{2}$ $\quad$ $-\frac{1}{2}\cos u \Big|_{\pi}^{3\pi}$

$-\frac{1}{2}\left[\cos(3\pi) - \cos(\pi)\right]$

$-\frac{1}{2}\left[-1 - -1\right]$

0

8. $$\int \frac{7x^3+4x^2+9x+4}{(x^2+1)^2}\,dx \rightarrow \frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$$

$$7x^3+4x^2+9x+4 = (Ax+B)(x^2+1)+Cx+D$$

$$7x^3+4x^2+9x+4 = Ax^3+Ax+Bx^2+B+Cx+D$$

$A=7$ $\quad B=4$ $\quad A+C=9$ $\quad B+D=4$

$(7)+C=9$ $\quad (4)+D=4$

$C=2$ $\quad D=0$

$$\int \frac{7x+4}{x^2+1}+\frac{2x}{(x^2+1)^2}\,dx$$

$$\int \frac{7x}{x^2+1}\,dx + \int \frac{4}{x^2+1}\,dx + \int \frac{2x}{(x^2+1)^2}\,dx$$

$u=x^2+1$ $\quad \int \frac{7\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$ $\quad u=x \quad a=1$ $\quad u=x^2+1$ $\quad \int \frac{\cancel{2x}}{u^2}\cdot\frac{du}{\cancel{2x}}$

$\frac{du}{dx}=2x$ $\quad du=dx$ $\quad \frac{du}{dx}=2x$

$dx=\frac{du}{2x}$ $\quad \frac{7}{2}\int \frac{1}{u}du$ $\quad 4\int \frac{1}{u^2+a^2}du$ $\quad dx=\frac{du}{2x}$ $\quad \int u^{-2}du$

$$\frac{7}{2}\ln|x^2+1| + 4\arctan x - \frac{1}{x^2+1}+C$$

9. $$\int \frac{x^2}{(x+2)^3}\,dx$$

$u=x+2$ $\quad \int \frac{x^2}{u^3}\,du$

$du=dx$ $\quad \int (u-2)^2\cdot u^{-3}\,du$

$$\int (u^2-4u+4)u^{-3}\,du$$

$$\int u^{-1}-4u^{-2}+4u^{-3}\,du$$

$$\ln|x+2| + \frac{4}{x+2} - \frac{2}{(x+2)^2}+C$$

10. $$\int \frac{4x^2\sin(2x^3)\cos(2x^3)}{1-\cos^2(2x^3)}\,dx$$

$$\int \frac{4x^2\cancel{\sin(2x^3)}\cos(2x^3)}{\sin^{\cancel{2}}(2x^3)}\,dx$$

$\int 4x^2\cot(2x^3)\,dx$ $\quad \int 4\cancel{x^2}\cdot\cot u\cdot\frac{du}{6\cancel{x^2}}$

$u=2x^3$ $\quad \frac{2}{3}\ln|\sin u|+C$

$\frac{du}{dx}=6x^2$ $\quad \frac{2}{3}\ln|\sin(2x^3)|+C$

$dx=\frac{du}{6x^2}$

11. $\int \frac{2x}{4x^4+5}\,dx$

$u = 2x^2 \quad a = \sqrt{5}$

$\frac{du}{dx} = 4x \qquad \int \frac{2x}{u^2+a^2} \cdot \frac{du}{4x}$

$dx = \frac{du}{4x} \qquad \frac{1}{2}\int \frac{1}{u^2+a^2}\,du$

$\frac{1}{2\sqrt{5}} \arctan \frac{2x^2}{\sqrt{5}} + C$

12. $\int x e^{-2x}\,dx$

$u = x \qquad \int dv = \int e^{-2x}\,dx$

$du = dx \qquad v = -\frac{1}{2}e^{-2x}$

$-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$

13. $\int \frac{3^{\ln(2x+1)}}{6x+3}\,dx$

$u = \ln(2x+1) \qquad \int \frac{3^u}{3(2x+1)} \cdot \frac{2x+1}{2}\,du$

$\frac{du}{dx} = \frac{2}{2x+1} \qquad \frac{1}{6}\int 3^u\,du$

$2dx = (2x+1)du \qquad \frac{1}{6} \cdot \frac{3^u}{\ln 3} + C$

$dx = \frac{2x+1}{2}du$

$\frac{3^{\ln(2x+1)}}{6\ln 3} + C$

14. $\int 3x\csc(2x^2)\,dx$

$u = 2x^2 \qquad \int 3\cancel{x}\csc u \cdot \frac{du}{4\cancel{x}}$

$\frac{du}{dx} = 4x \qquad \frac{3}{4}\int \csc u\,du$

$dx = \frac{du}{4x} \qquad -\frac{3}{4}\ln|\csc u + \cot u| + C$

$-\frac{3}{4}\ln|\csc(2x^2) + \cot(2x^2)| + C$

15. $\int \frac{5x}{\sqrt{25-2x^2}}\,dx$

$u = 25 - 2x^2$ $\qquad \int \frac{5\cancel{x}}{u^{1/2}} \cdot \frac{du}{-4\cancel{x}}$

$\frac{du}{dx} = -4x$ $\qquad -\frac{5}{4}\int u^{-1/2}\,du$

$dx = \frac{du}{-4x}$ $\qquad -\frac{5}{2}u^{1/2} + C$

$-\frac{5}{2}(25-2x^2)^{1/2} + C$

16. $\int x(x^2+1)^{3/2}\,dx$

$u = x^2 + 1$ $\qquad \int \cancel{x} \cdot u^{3/2} \cdot \frac{du}{2x}$

$\frac{du}{dx} = 2x$ $\qquad \frac{1}{2}\int u^{3/2}\,du$

$dx = \frac{du}{2x}$ $\qquad \frac{1}{5}u^{5/2} + C$

$\frac{1}{5}(x^2+1)^{5/2} + C$

17. $\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx \longrightarrow \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$

$x = 0:$ $\qquad x = -1:$ $\qquad A + B = 5$

$6 = A$ $\qquad -9 = -C$ $\qquad B = -1$

$C = 9$

$\int \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\,dx$

$u = x+1$ $\qquad 9\int u^{-2}\,du$

$du = dx$ $\qquad -9u^{-1}$

$6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$

18. $\int \arcsin 3x \, dx$

$u = \arcsin 3x \qquad \int dv = \int dx$

$\frac{du}{dx} = \frac{3}{\sqrt{1-9x^2}} \qquad v = x$

$x \arcsin 3x - \int \frac{3x}{\sqrt{1-9x^2}} dx$

$u = 1-9x^2 \qquad \int \frac{3\cancel{x}}{u^{1/2}} \cdot \frac{du}{-18\cancel{x}}$

$\frac{du}{dx} = -18x$

$dx = \frac{du}{-18x} \qquad -\frac{1}{6}\int u^{-1/2} du$

$-\frac{1}{3} u^{1/2}$

$x \arcsin 3x + \frac{1}{3}(1-9x^2)^{1/2} + C$

19. $\int_0^1 x e^{x^2} dx$

$u = x^2 \qquad \int_0^1 \cancel{x} e^u \cdot \frac{du}{2\cancel{x}}$

$\frac{du}{dx} = 2x \qquad \frac{1}{2}\int_0^1 e^u du$

$dx = \frac{du}{2x} \qquad \frac{1}{2}\left[e^u \Big|_0^1\right]$

$\frac{1}{2}\left[e^1 - e^0\right]$

$\frac{1}{2}\left[e-1\right]$

20. $\frac{d}{dx}\int_2^{x^2} (1-t^3) dt$

$\left[1-(x^2)^3\right](2x)$

$2x(1-x^6)$